List of Comprehensive Exams Topics

Mechanics

1. Basic Mechanics
    - Newton's laws and conservation laws, the virial theorem

2. The Lagrangian and Hamiltonian Formalism
    - The Lagrange formalism and the principle of least action principle
    - The Hamiltonian formalism
    - Poisson brackets
    - Symmetries and conservation laws in the Lagrangian and Hamiltonian formalisms. Noether theorem.

3. Some Basic Problems
    - One-dimensional and 1D-reducible problems
    - Equilibrium and statics
    - Planetary motion and Kepler's laws
    - Elastic scattering and the Rutherford formula
    - Motion of (relativistic) particles in external electromagnetic fields
    - Adiabatic invariance

4. Oscillations
    - Free and forced oscillations
    - Weakly nonlinear oscillations and associated approximation scheme
    - Parametric resonance and excitation

5. From Oscillations to Waves
    - Coupled oscillators
    - 1D waves. Periodic systems. Reflections at interfaces and boundaries

6. Rigid Body Motion
    - Angular velocity vector and the moment of inertia tensor; fixed-axis rotation
    - Tops, free rotation, Euler equations, precession
    - Non-inertial reference frames

7. Fluid Mechanics
    - Equations of motion for ideal and viscous fluids. Compressible and incompressible fluids. Reynolds number. Hydrostatics.

Classical Electrodynamics

1. Fundamental Electrostatics
    - The electrostatic field and potential, Gauss' law
    - Electric field energy and the electrostatic stress tensor
    - Capacitance
    - Method of images
    - Separation of variables and Green functions in cartesian, spherical, and cylindrical coordinates

2. Polarization and Dielectrics
    - The multipole expansion and electric dipoles
    - Dielectric media and boundary conditions

3. DC Currents
    - Continuity equation, Kirchhoff laws, and Ohm's law
    - Energy dissipation rate in Ohmic materials

4. Magnetism and Magnetic Materials
    - Magnetic interaction of currents
    - Vector-potential and Ampère's law
    - Magnetic flux, energy, and inductance
    - Magnetic dipoles
    - Magnetic media and boundary conditions
    - The magnetic stress tensor

5. Time-Dependent Electromagnetism
    - Electromagnetic induction
    - Self and mutual inductances
    - Quasistatic approximation and the skin effect
    - Gauge invariance in classical electrodynamics
    - Displacement current
    - Inductors and AC circuits at an elementary level (i.e. LR, LC, LRC circuits)

6. Electromagnetic Wave Propagation
    - Plane waves: velocity, power
    - Waves in media
    - Dissipation and dispersion, Kramers Kronig relations
    - Reflection and refraction
    - Waves in waveguides, resonant cavities

7. Radiation, Scattering, Interference, and Diffraction
    - Near zone vs. radiation zone
    - Multipole radiation (electric, magnetic, and quadrupole radiation)
    - Radiation spectrum
    - Basics of scattering: dipole scattering and light electron scattering
    - Born approximation, Fresnel and Fraunhofer diffraction

8. Special Relativity
    - Relativistic kinematics (time dilation, length contraction, simultaneity), momentum, mass, and energy
    - Maxwell equations in covariant form
    - Transformation of field strengths and the electromagnetic stress tensor
    - The action, Lagrangian, and Hamiltonian of particles in electromagnetic fields.
    - The action, Lagrangian, and Hamiltonian of the electromagnetic field

9. Radiation by Relativistic Charges
    - Liénard-Wiechert potentials
    - Radiation fields and radiated energy from relativistic particles
    - Synchrotron radiation
    - Radiation during collisions
    - Photons and the limitations of classical electrodynamics

Quantum Mechanics

1. Formalism of Quantum Mechanics
    - Continuity equation
    - States, state vectors, and linear operators
    - State basis and matrix representation
    - Change of basis, unitary operators, and matrix diagonalization
    - Observables: expectation values and Ehrenfest's theorem, uncertainties, and uncertainty relations, virial theorem
    - Quantum dynamics: the Schrodinger, Heisenberg, and interaction pictures

2. 1D Wave Mechanics
    - Free particle: Wave packets
    - Motion in simple potential profiles, harmonic oscillator
    - The WKB approximation
    - Transfer matrix, resonant tunneling, and metastable states
    - Coupled quantum wells
    - 1D band theory, effective mass, Bloch oscillations, Landau-Zener tunneling

3. 3D Wave Mechanics
    - Minimal substitution, the Aharonov-Bohm effect and magnetic flux quantization
    - Landau levels and the quantum Hall effect
    - Scattering and diffraction
    - Axially-symmetric systems
    - Spherically-symmetric systems, the isotropic harmonic oscillator
    - Atoms and the periodic table

4. Some Exactly Solvable Time Dependent Problems
    - Two-level (spin-1⁄2 systems) systems, Bell's theorem
    - Feynman's path integral for quadratic Lagrangians
    - Coherent (or Glauber) states

5. Symmetries in Quantum Mechanics and Angular Momentum
    - Translational symmetry and linear momentum.
    - Angular momentum, rotational symmetry, and spherically-symmetric problems
    - Spin and its addition to orbital angular momentum. Clebsch-Gordan coefficients
    - Parity symmetry. Time reversal and anti-unitary operators. Kramers degeneracy
    - Gauge symmetry and gauge invariance in nonrelativistic quantum mechanics

6. Approximation Methods
    - Variational method

- Eigenvalue and eigenstate problems in a perturbative approximation. Examples: the linear and quadratic Stark effects, fine structure of atomic levels, the Zeeman effect
- Time-dependent perturbations; the quantum-mechanical Golden Rule for time dependent and step-like perturbations
- The adiabatic approximation

7. Open Quantum Systems
   - Mixed states and the density matrix representation
   - Density matrix dynamics, dissipation, dephasing, and relaxation

8. Multiparticle Systems
   - Distinguishable and indistinguishable particles
   - Singlets, triplets, and the exchange interaction
   - Second quantization

9. Interaction with the Quantized Electromagnetic Field
   - Electromagnetic field quantization
   - Spontaneous and stimulated emission
   - The photoelectric effect and photocount statistics

10. Basics of Relativistic Quantum Mechanics
    - The Dirac equation and its plane wave solutions

Statistical Mechanics

1. Thermodynamics
    - The laws of thermodynamics, entropy, temperature, heat capacity
    - Thermodynamic potentials
    - Systems with variable number of particles
    - Thermal Machines (refrigerators, engines, etc)
    - Examples: Clausius-Clapeyron equation, throttling process

2. Principles of Physical Statistics
    - Statistical ensemble and probability
    - Microcanonical ensemble and distribution
    - Canonical ensemble and the Gibbs distribution
    - Harmonic oscillator statistics and its major applications
    - Grand canonical ensemble and distribution
    - Systems of independent quantum particles

3. Ideal and Not-So-Ideal Gases
    - Ideal classical gas and its chemical potential
    - Degenerate Fermi gas
    - Bose-Einstein condensation
    - Gases of weakly interacting particles
    - First order phase transitions; the van der Waals equation of state

4. Continuous Phase Transitions
    - Basic notions of continuous phase transitions
    - Landau's mean-field theory
    - Ising model: Weiss mean field theory
    - Ising model in 1D, critical exponents, and qualitative features in 2D

5. Fluctuations
    - Characterization of fluctuations
    - Fluctuations of energy, the number of particles, volume, and temperature
    - Fluctuations as functions of time; the correlation function and the spectral density
    - Fluctuations and dissipation, Green-Kubo formula
    - Brownian motion, the diffusion equation, and the Fokker-Planck equation

6. Elements of Kinetics
    - The Liouville theorem
    - The Boltzmann equation and the relaxation-time approximation.
    - The Ohm law and the Drude formula

- Electrochemical potential and drift-diffusion equation