AMS Foundation Exam (January 2023): Probability Questions

Date of Exam: January 19, 2023

Time: 11:15 am - 1:15 pm

There are 3 problems, and you are required to solve all of them. All problems are weighted equally. Please show detailed work for full credit.

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination.

NAME: _____ **ID:** _____

Signature: _____

1. There are four six-sided dice. One of them has exactly one side painted white, and each of the remaining three dice has exactly three sides painted white. One of these four dice was chosen randomly and tossed 7 times. A white side appeared exactly once. Find the probability that the chosen die has only one side painted white.

2. Let A and B be independent random variables with uniform distributions on the interval $[0, 2]$. Find the probability that there exists a real number x such that $x^2 + 2Ax + 2B = 0$.

3. For two random variables X and Y with a joint cumulative distribution function F satisfying $F(x, y) = \sin x \sin y$ for $0 \leq x \leq \pi/2$ and $0 \leq y \leq \pi/2$, compute the following: (i) the probability density function $f(x, y)$, (ii) vector of expectations $(E[X], E[Y])$, (iii) vector of variances (σ_X^2, σ_Y^2), and (iv) covariance $\mathrm{Cov}(X, Y)$. In addition, find the entire function $F(x, y)$ for all $(x, y) \in \mathbb{R}^2$.