

AMS Foundation Exam (January 2020): Probability Questions

All problems are weighted equally. On this cover page write which three problems you want graded.

problems to be graded (circle 3): 1 2 3 4

Name (PRINT CLEARLY): _____

ID: _____

Instruction: Please show detailed work for full credit. This is a closed book exam. Please do NOT use a calculator or cell phone.

Date of Exam: January 27, 2020

Time: 11:15 am - 1:15 pm

1. A child is missing in an amusement park, and it is presumed that she is equally likely to be in any of 4 possible areas. Let γ_i denote the probability that the child will be found upon a search of the i th area when she is, in fact, in that area, $i = 1, 2, 3, 4$. What is the conditional probability that the child is in the i th area, given that a search of area 1 is unsuccessful, $i = 1, 2, 3, 4$?

2. Let X be a nonnegative random variable. Prove that

$$E(X) \leq [E(X^2)]^{1/2} \leq [E(X^3)]^{1/3} \leq \dots$$

3. The joint density of X and Y is given by

$$f(x, y) = C(y - x)e^{-y}, \quad -y < x < y, \quad 0 < y < \infty.$$

- (a) Find C .
- (b) Find the marginal density function of X .
- (c) Find the marginal density function of Y .
- (d) Find EX .
- (e) Find EY .
4. Suppose there are n cars driving forward in a straight line. No car can pass another car in front, so we can label the cars by $1, \dots, n$ where the car labeled i is the i th car from the front. Let v_1, \dots, v_n be distinct positive real numbers, and suppose that the i th car goes at constant speed v_i unless it is directly behind the $(i - 1)$ th car in which case it goes at the speed of the car in front. Eventually, the cars will cluster in groups where each group is a cluster of faster cars stuck behind slower cars. For example, if $v_1 = 3, v_2 = 1, v_3 = 2$, then car 1 will drive ahead while cars 2 and 3 will be together in one group. Assume that v_1, \dots, v_n is a random permutation of positive real numbers $a_1 < a_2 < \dots < a_n$. What is the expected number of groups as a function of n after allowing sufficient time to form the final groups?